C & EE 141

Bending Members

Laterally Unsupported and Noncompact

Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

- *Only limited to these three limit states when compression flange is laterally braced and beam is compact.*

What if the compression flange isn't braced?
What if there is a local buckling issue?

Limit States for Flexure

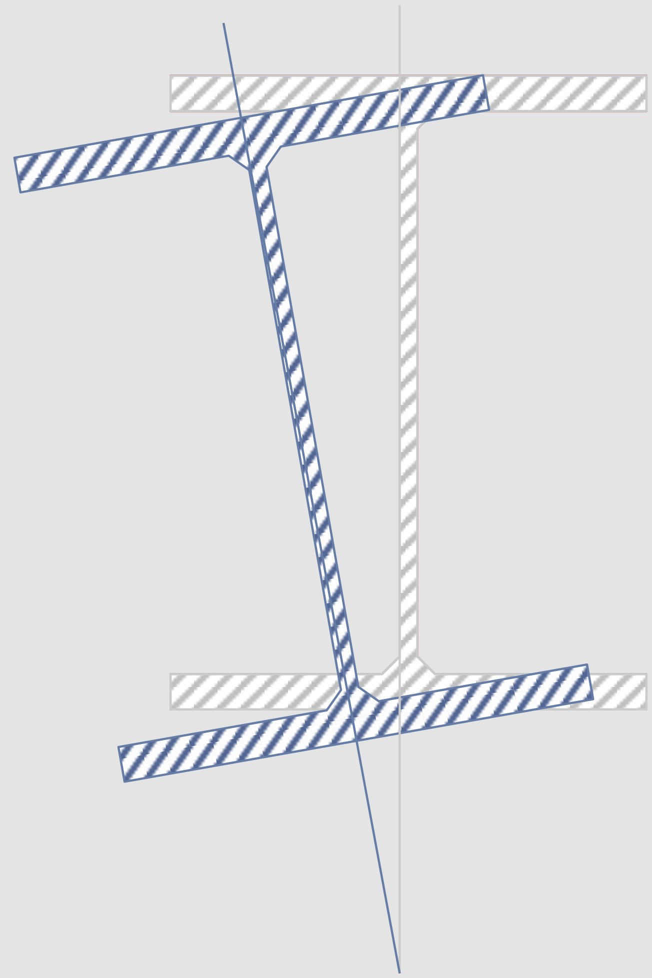
- Plastic Flexural Capacity
- **Global Buckling**
  - **Inelastic lateral torsional buckling**
  - **Elastic lateral torsional buckling**
- **Local Buckling**
  - **Width-thickness ratios for web and**
- Shear Capacity
- Deflection

Limit States for Flexure

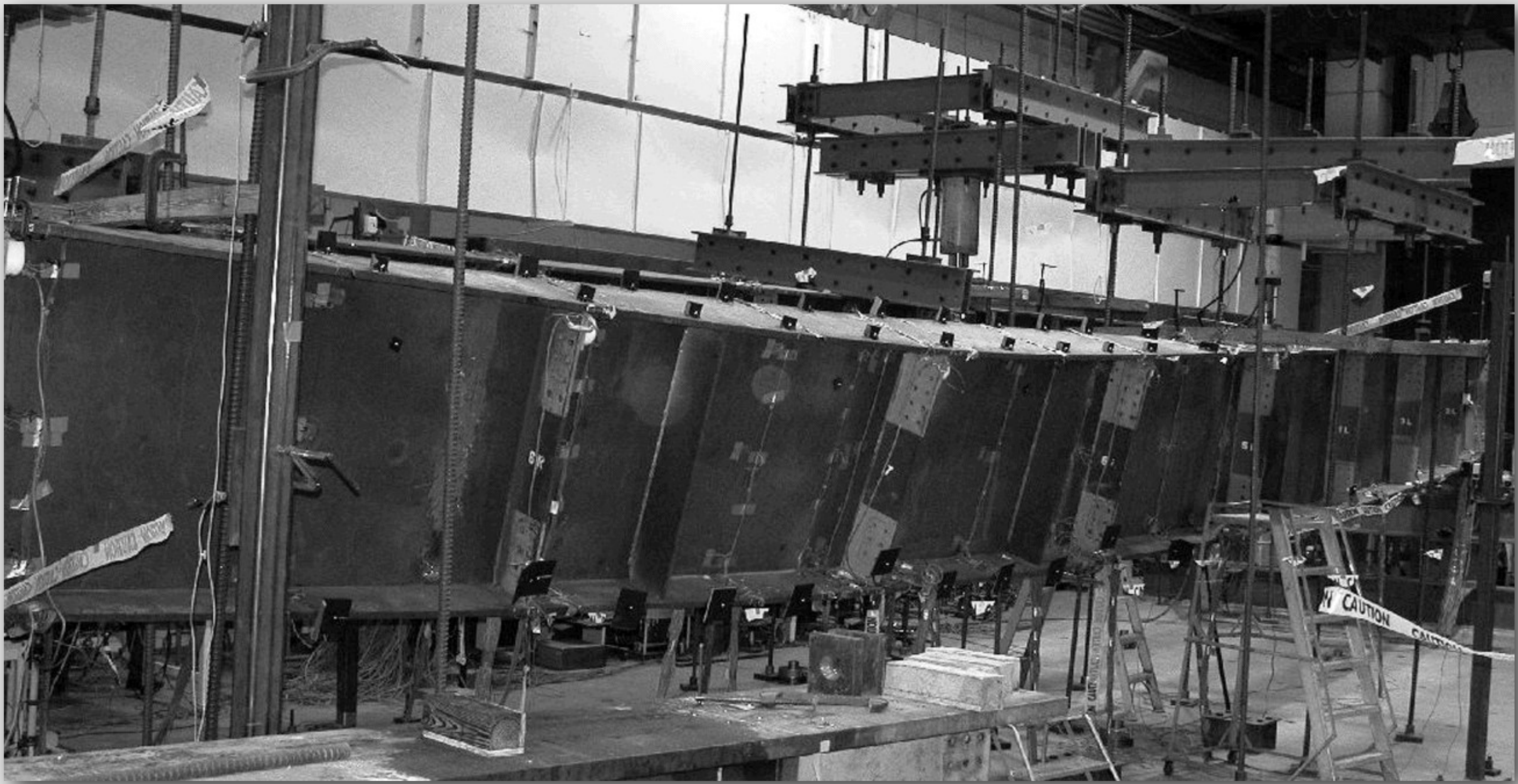
- Plastic Flexural Capacity
- Global Buckling
  - Inelastic lateral torsional buckling
  - Elastic lateral torsional buckling
- Local Buckling
  - Width-thickness ratios for web and flanges
- Shear Capacity
- Deflection

What is Lateral Torsional Buckling?

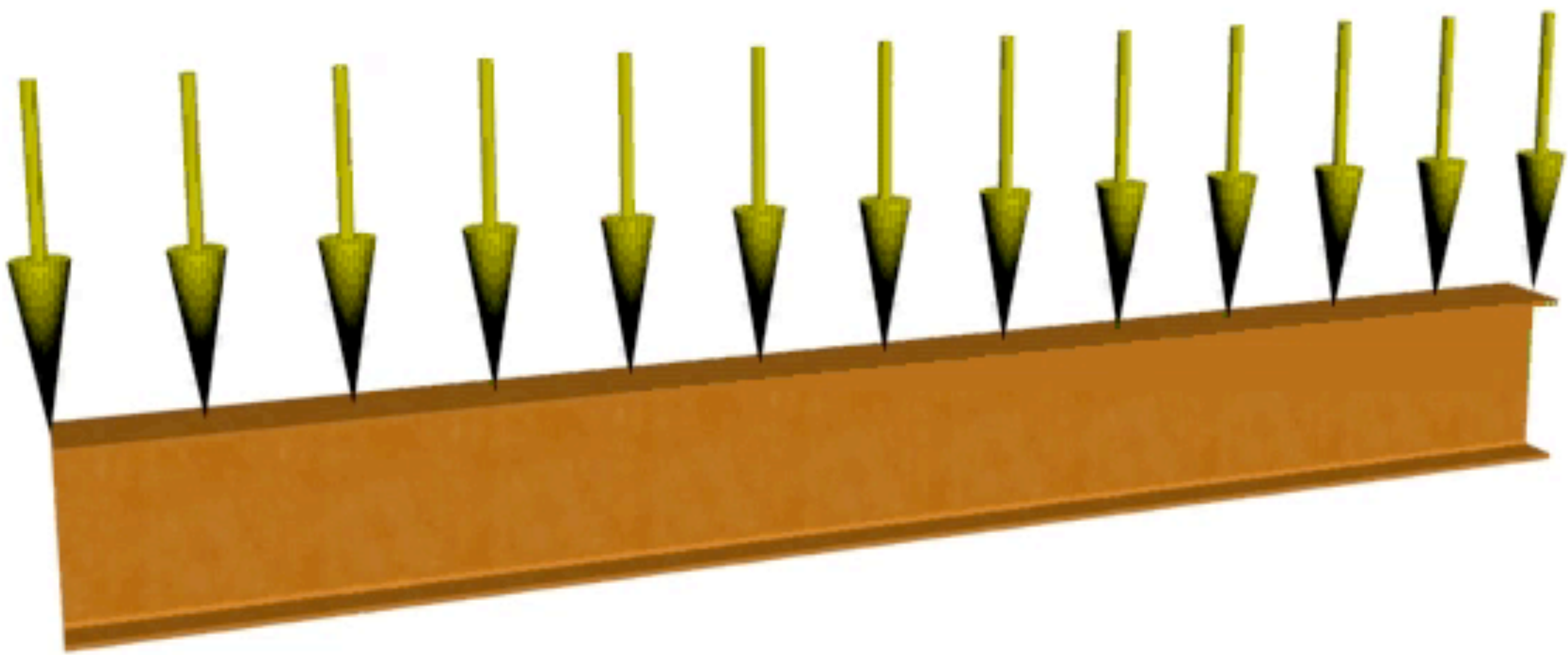
- The compression zone of the section buckles if not restrained
- For a simply supported WF that is the top flange and, to a varying extent, the web
- For a cantilevered WF that is the bottom flange and, to a varying extent, the web



What is Lateral Torsional Buckling?

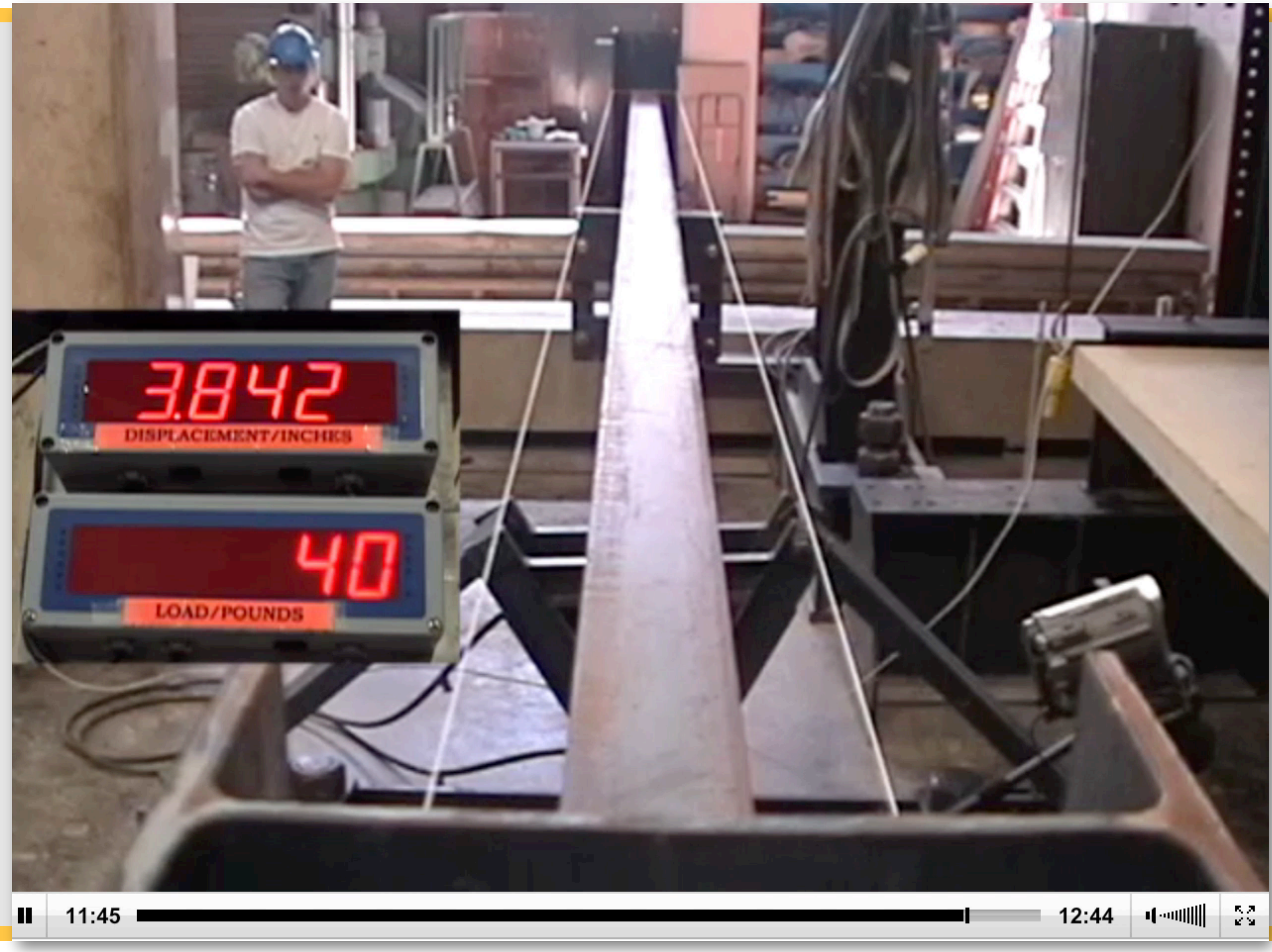


Beams experience Lateral Torsional Buckling (LTB), due to buckling of the compression flange of a beam



Onset of Lateral Torsional Buckling

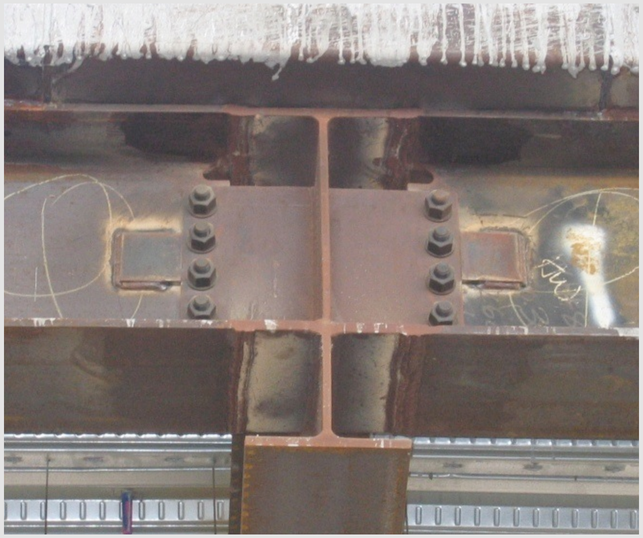
Elastic LTB



How is Lateral Torsional Buckling Prevented?

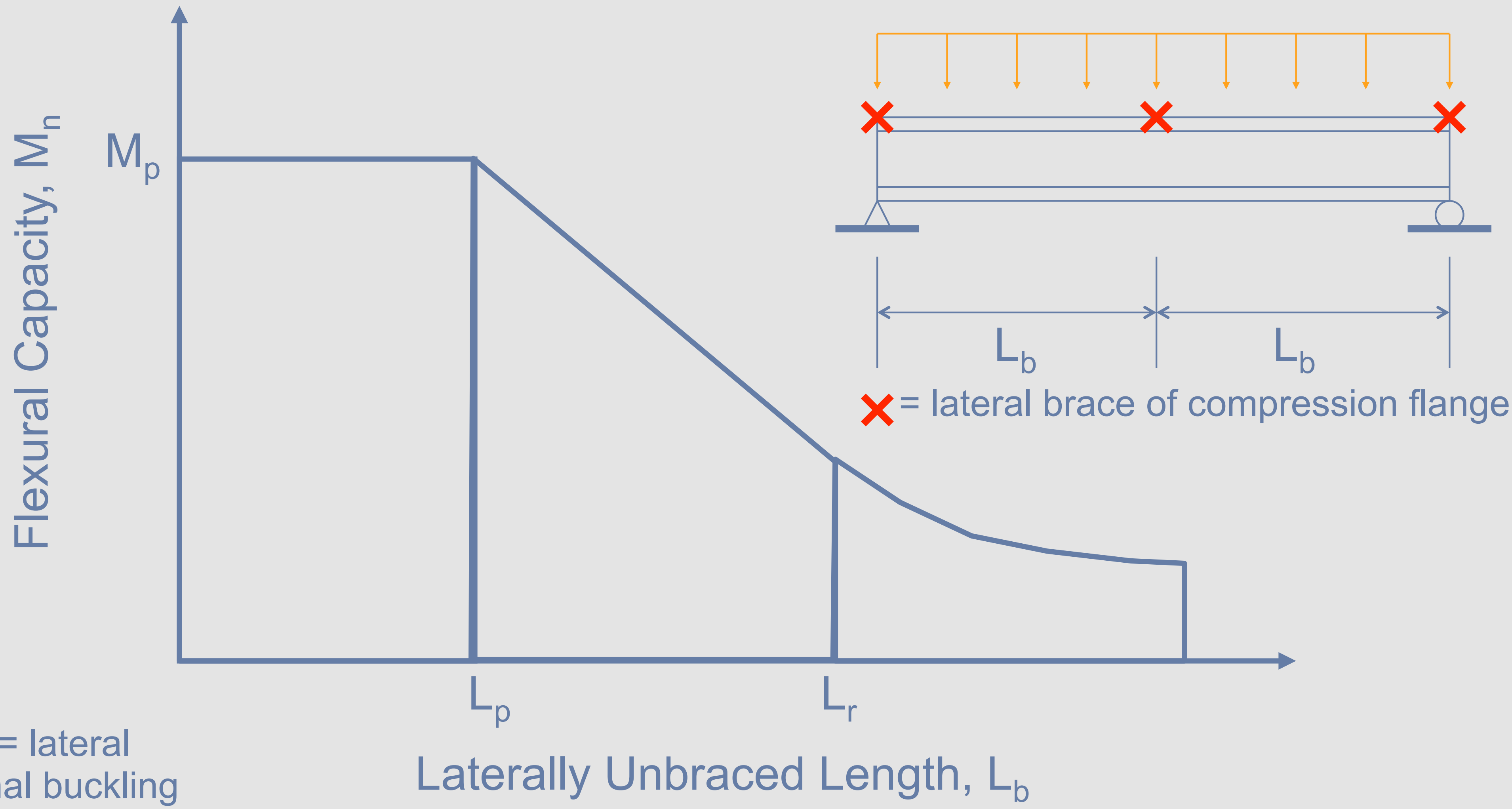
- Headed steel studs welded to the beam flange harden in the concrete fill which provides continuous stability

- Connections from perpendicular framing beams provide discrete points of stability



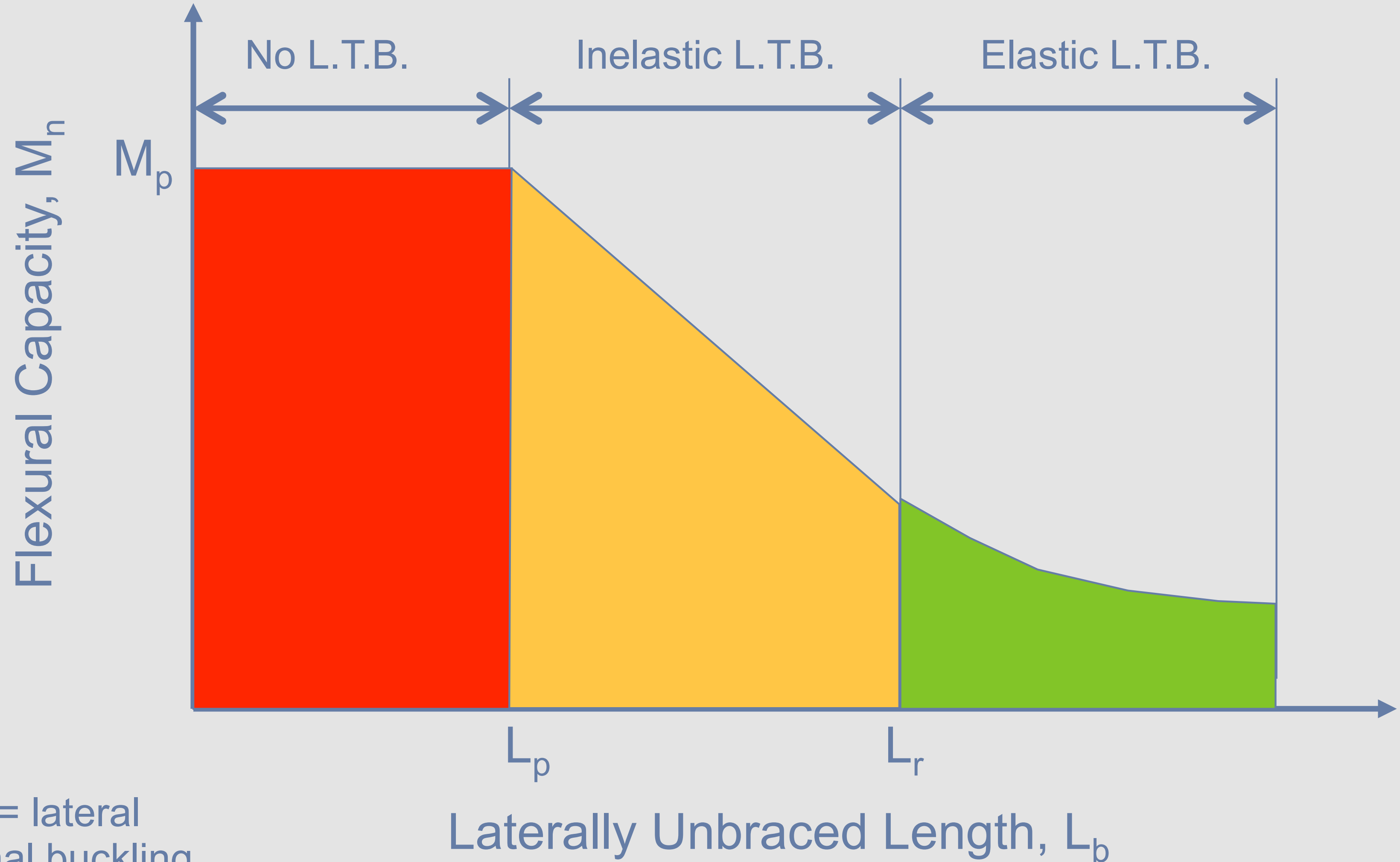
Example of an Unbraced Beam

Transitions in Flexural Capacity due to Global Buckling



L.T.B. = lateral torsional buckling

Transitions in Flexural Capacity due to Global Buckling



L.T.B. = lateral torsional buckling

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric I-shaped members and channels bent about their major axis, having compact webs and compact flanges as defined in Section B4.1 for flexure.

> **User Note:** All current ASTM A6 W, S, M, C and MC shapes except W21×48, W14×99, W14×90, W12×65, W10×12, W8×31, W8×10, W6×15, W6×9, W6×8.5 and M4×6 have compact flanges for F_y = 50 ksi (345 MPa); all current ASTM A6 W, S, M, HP, C and MC shapes have compact webs at $F_y \leq$ 65 ksi (450 MPa).

The *nominal flexural strength*, M_n, shall be the lower value obtained according to the *limit states* of *yielding (plastic moment)* and *lateral-torsional buckling*.

1. Yielding

$$M_n = M_p = F_y Z_x \tag{F2-1}$$

where

F_y = *specified minimum yield stress* of the type of steel being used, ksi (MPa)

Z_x = plastic section modulus about the x-axis, in.^{3} (mm^3)

2. Lateral-Torsional Buckling

(a) When $L_b \le L_p$, the *limit state* of *lateral-torsional buckling* does not apply.

(b) When $L_p < L_b \le L_r$

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(F2-2)}$$

(c) When $L_b > L_r$

$$M_n = F_{cr}S_x \le M_p \qquad \text{(F2-3)}$$

where

L_b = length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section, in. (mm)

$$F_{cr} = \frac{C_b\pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_o}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(F2-4)}$$

and where

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)
J = torsional constant, in.^{4} (mm^4)
S_x = elastic section modulus taken about the x-axis, in.^{3} (mm^3)
h_o = distance between the flange centroids, in. (mm)

The limiting lengths L_p and L_r are determined as follows:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \tag{F2-5}$$

$$\boxed{L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}} \tag{F2-6}$$

where

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \tag{F2-7}$$

and the coefficient c is determined as follows:

(a) For doubly symmetric I-shapes: $c = 1$ (F2-8a)

(b) For channels: $c = \frac{h_o}{2}\sqrt{\frac{I_y}{C_w}}$ (F2-8b)

User Note: The square root term in Equation F2-4 may be conservatively taken equal to 1.0.

User Note: Equations F2-3 and F2-4 provide identical solutions to the following expression for lateral-torsional buckling of doubly symmetric sections that has been presented in past editions of the AISC LRFD Specification:

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$

The advantage of Equations F2-3 and F2-4 is that the form is very similar to the expression for lateral-torsional buckling of singly symmetric sections given in Equations F4-4 and F4-5.

User Note: For doubly symmetric I-shapes with rectangular flanges, $C_w = \frac{I_y h_o^2}{4}$ and thus Equation F2-7 becomes

$$r_{ts}^2 = \frac{I_y h_o}{2S_x}$$

r_{ts} may be approximated accurately and conservatively as the radius of gyration of the compression flange plus one-sixth of the web:

$$r_{ts} = \frac{b_f}{\sqrt{12\left(1 + \frac{1}{6}\frac{ht_w}{b_f t_f}\right)}}$$

AISC CH16-F

Global Buckling Limits

1. Inelastic LTB ($L_p < L_b \leq L_r$)

$$M_n = C_b \left[M_p - \left(M_p - 0.7 F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad \text{(Eq F2-2)}$$

$$\phi_b M_n = C_b \left[\phi_b M_p - \phi_b BF \left(L_b - L_p \right) \right] \leq \phi_b M_p \quad \text{(pg 3-9)}$$

Table 3-2

2. Elastic LTB ($L_b > L_r$)

$$M_n = F_{cr} S_x \leq M_p \quad \text{(Eq F2-3)}$$

where F_{cr} per Eq F2-4

F_y = 50 ksi

Table 3-2 (continued)
W-Shapes
Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W30×116	**378**	**943**	**1420**	**575**	**864**	**24.8**	**37.4**	**7.74**	**22.6**	**4930**	**339**	**509**
W21×147	373	931	1400	575	864	13.7	20.7	10.4	36.3	3630	318	477
W24×131	370	923	1390	575	864	16.3	24.6	10.5	31.9	4020	296	445
W18×158	356	888	1340	541	814	10.5	15.9	9.68	42.8	3060	319	479
W14×193	355	886	1330	541	814	5.30	7.93	14.3	79.4	2400	276	414
W12×210	348	868	1310	510	767	4.25	6.45	11.6	95.8	2140	347	520
W30×108	**346**	**863**	**1300**	**522**	**785**	**23.5**	**35.5**	**7.59**	**22.1**	**4470**	**325**	**487**
W27×114	343	856	1290	522	785	21.7	32.8	7.70	23.1	4080	311	467
W21×132	333	831	1250	515	774	13.2	19.9	10.3	34.2	3220	283	425
W24×117	327	816	1230	508	764	15.4	23.3	10.4	30.4	3540	267	401
W18×143	322	803	1210	493	740	10.3	15.7	9.61	39.6	2750	285	427
W14×176	320	798	1200	491	738	5.20	7.83	14.2	73.2	2140	252	378

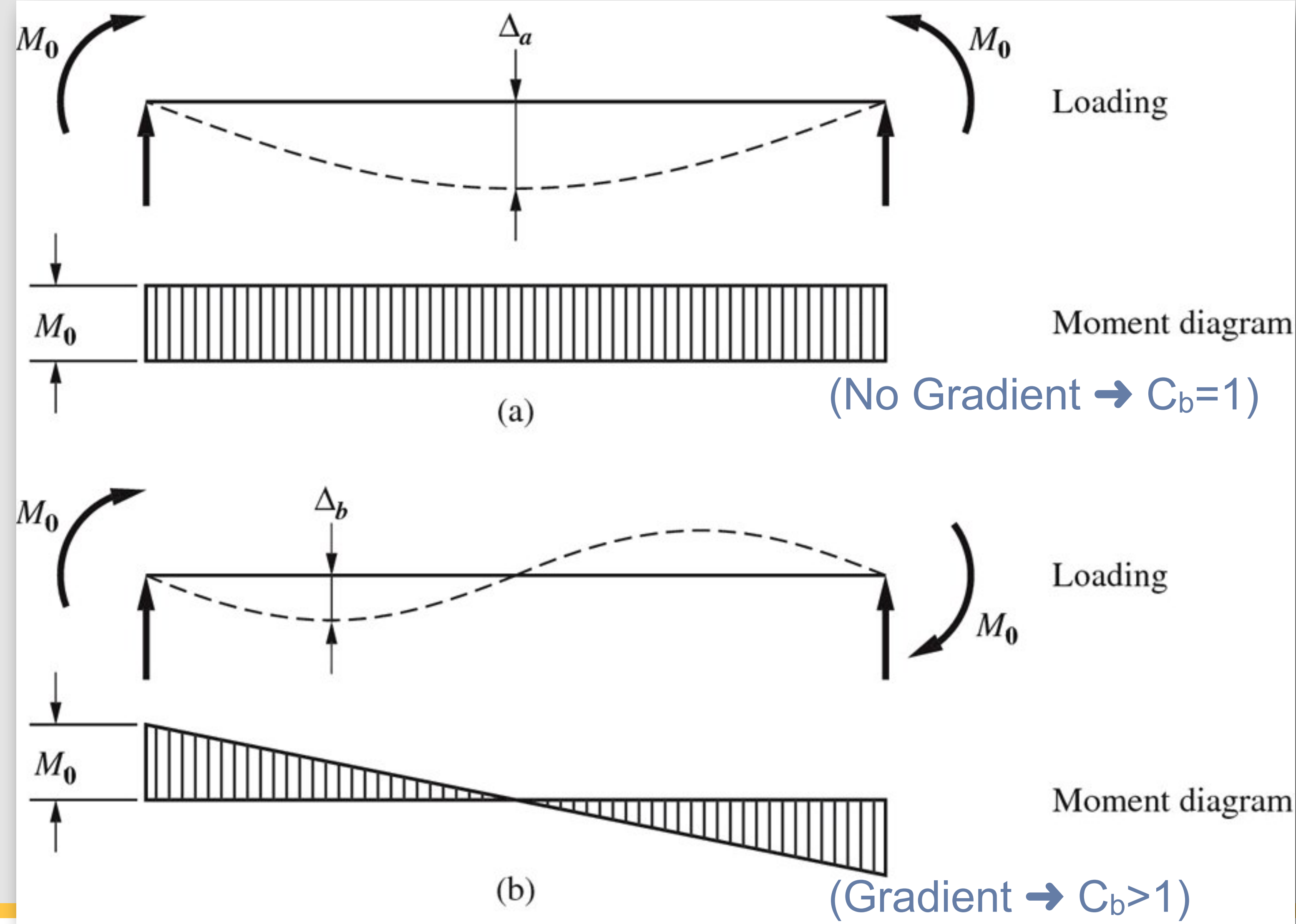
ASD	LRFD
$\Omega_b = 1.67$	$\phi_b = 0.90$
$\Omega_v = 1.50$	$\phi_v = 1.00$

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with $F_y = 50$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

The Moment Gradient (C_b)

- C_b is a coefficient used to account for the effect of moment gradients on LTB
- End restraints change the moment diagram of the beam so that the effective length of the “compression” element of a beam may change
- C_b is needed because all calculations and tables are based on a case with uniform moment, where $C_b = 1.0$

The Moment Gradient (C_b)



(No Gradient ➜ C_b=1)

(Gradient ➜ C_b>1)

The Moment Gradient (C_b)

(3) For singly symmetric members in *single curvature* and all doubly symmetric members:

C_b, the *lateral-torsional buckling* modification factor for nonuniform moment diagrams when both ends of the segment are braced is determined as follows:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad \text{(F1-1)}$$

where

M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. (N-mm)

M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. (N-mm)

M_B = absolute value of moment at centerline of the unbraced segment, kip-in. (N-mm)

M_C = absolute value of moment at three-quarter point of the unbraced segment, kip-in. (N-mm)

For cantilevers or overhangs where the free end is unbraced, C_b = 1.0.

The Moment Gradient (C_b)

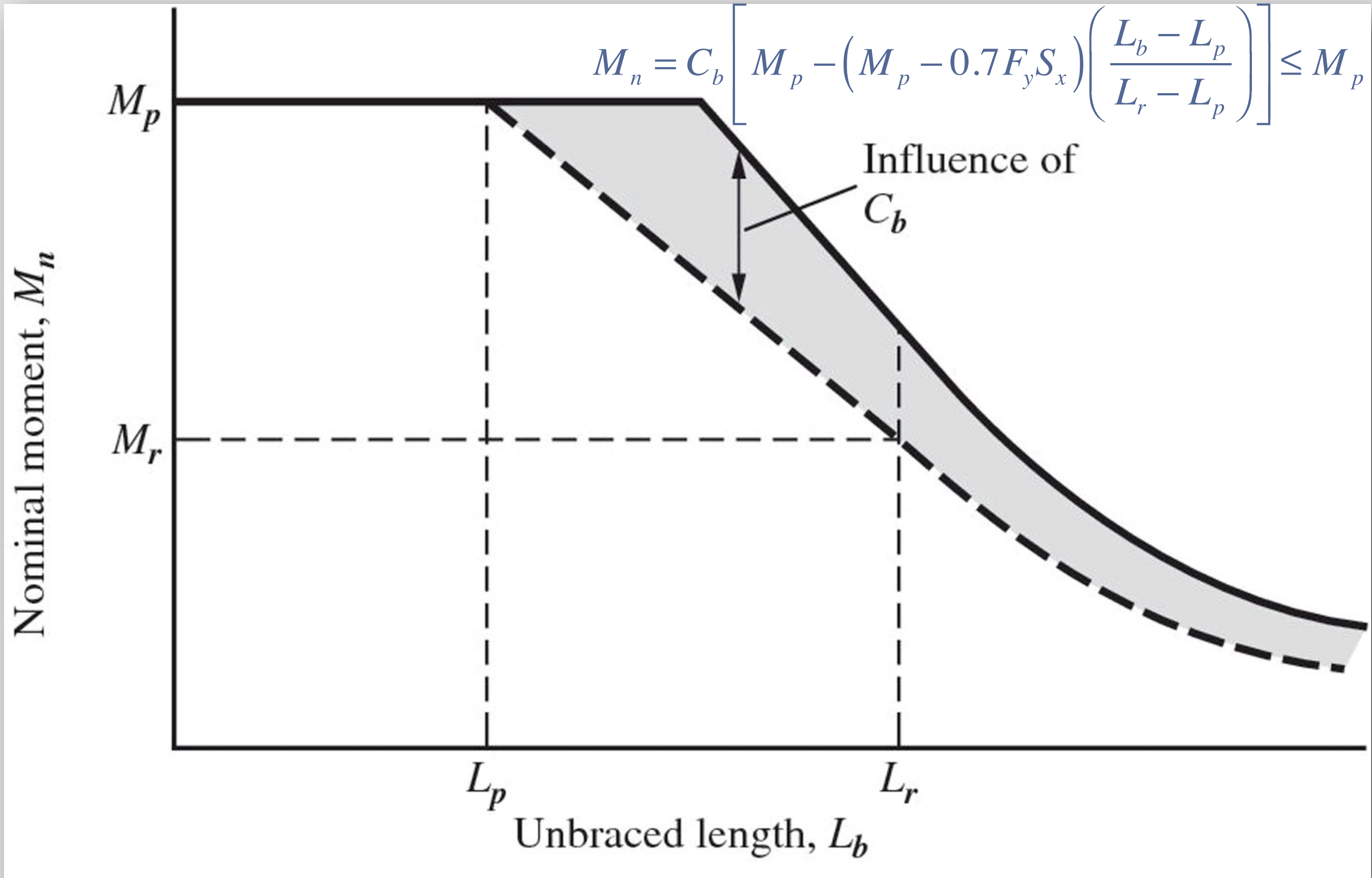
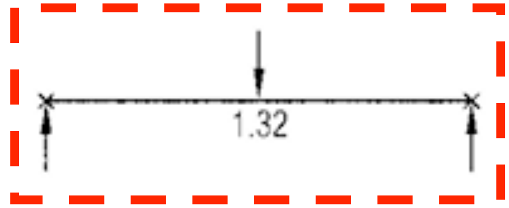


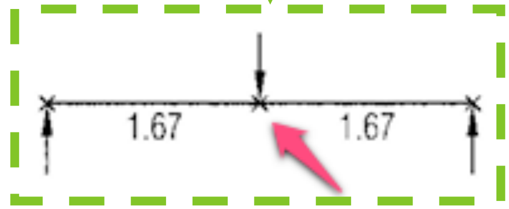
Figure 6.14

Table 3-1
Values of C_b for Simply Supported Beams

Load	Lateral Bracing Along Span	C_b
P	None Load at midpoint	1.32
	At load point	1.67 1.67
P P	None Loads at third points	1.14
	At load points Loads symmetrically placed	1.67 1.00 1.67
P P P	None Loads at quarter points	1.14
	At load points Loads at quarter points	1.67 1.11 1.11 1.67
w	None	1.14
	At midpoint	1.30 1.30
	At third points	1.45 1.01 1.45
	At quarter points	1.52 1.06 1.06 1.52
	At fifth points	1.56 1.12 1.00 1.12 1.56

Note: Lateral bracing must always be provided at points of support per AISC *Specification* Chapter F.





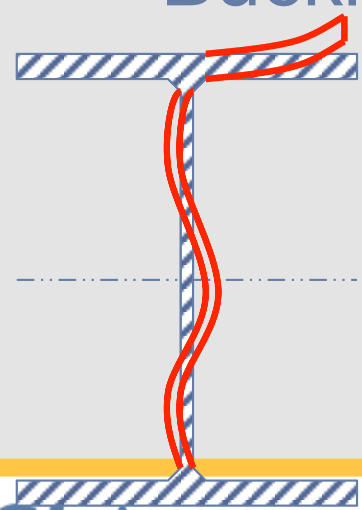
The little "x" indicates a lateral bracing point

Limit States for Flexure

- Plastic Flexural Capacity
- Global Buckling
  - Inelastic lateral torsional buckling
  - Elastic lateral torsional buckling
- Local Buckling
  - Width-thickness ratios for web and flanges
- Shear Capacity
- Deflection

Local Instability

- Buckling of one of the compression elements of the x–section.
  - Flange Local Buckling
  - Web Local Buckling



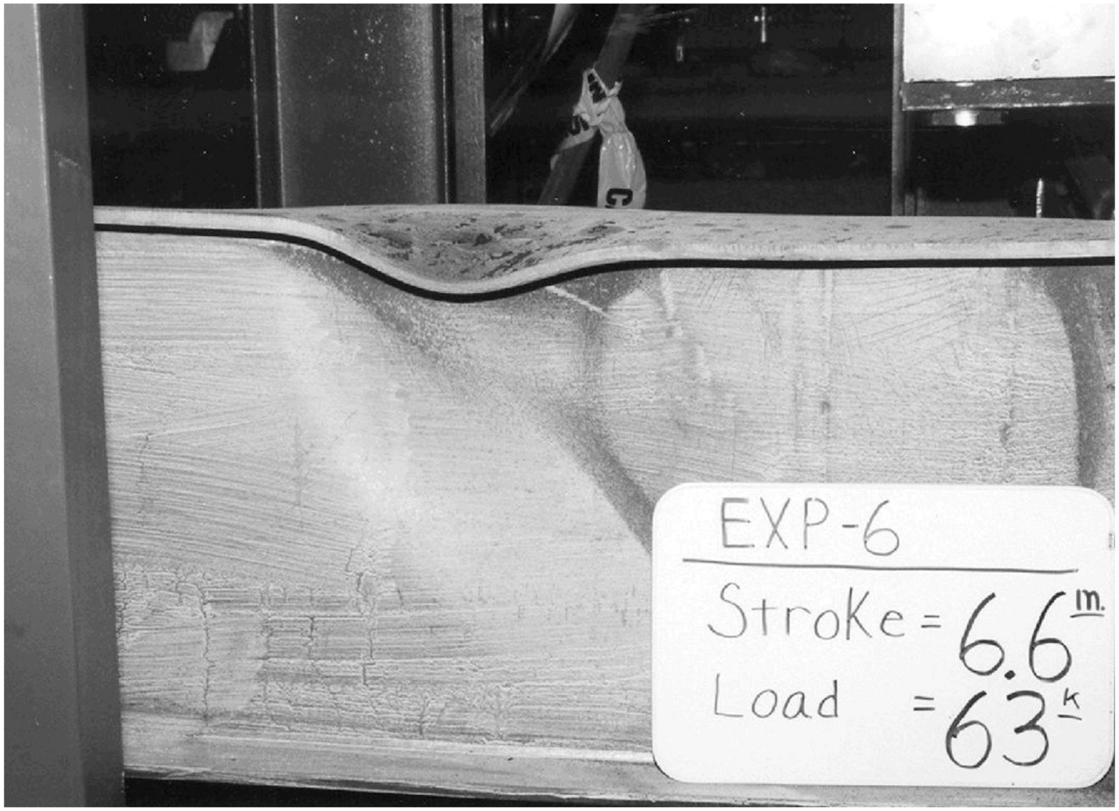


Figure 6.16
Photo courtesy of Donald W. White

Local Instability

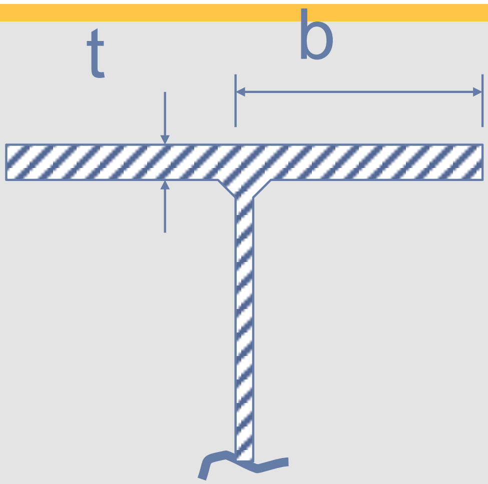
- Whether a member is subject to a local instability is a function of the width-thickness ratio of the elements that can buckle:

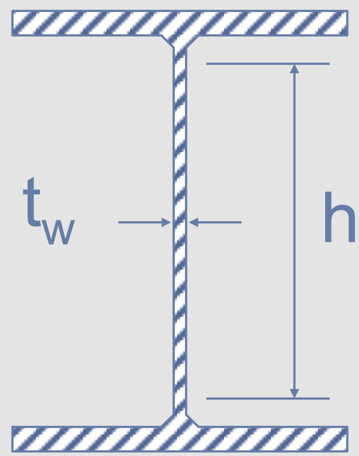
  - Flange Local Buckling

$$\lambda = b/t$$

  - Web Local Buckling

$$\lambda = h/t_w$$

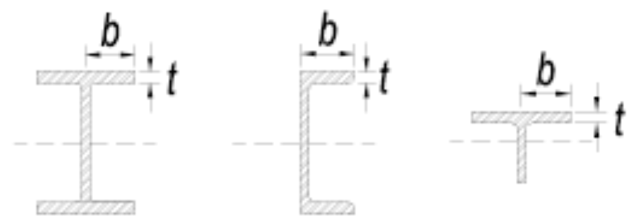
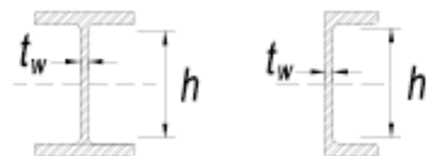




Checking the Ratios

- Limiting width-thickness ratios are function of:
  - geometry of the shapes
  - yield stress of the steel
- Shapes can be classified as:
  - A **compact shape** is not subject to local buckling.
  - A **non-compact shape** is subject to inelastic local buckling after initial yielding.
  - A **slender shape** is subject to elastic L.B.
- Local buckling will limit the moment capacity that can be reached by the member, regardless of L_b

Classification of Sections for Local Buckling

Case	Description of Element	(λ) Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio: λ_p (compact/noncompact)	Limiting Width-to-Thickness Ratio: λ_r (noncompact/slender)	Examples
10	Flanges of rolled I-shaped sections, channels, and tees	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
15	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	

Spec Table B4.1b

Effect on Flexural Capacity due to Local Buckling

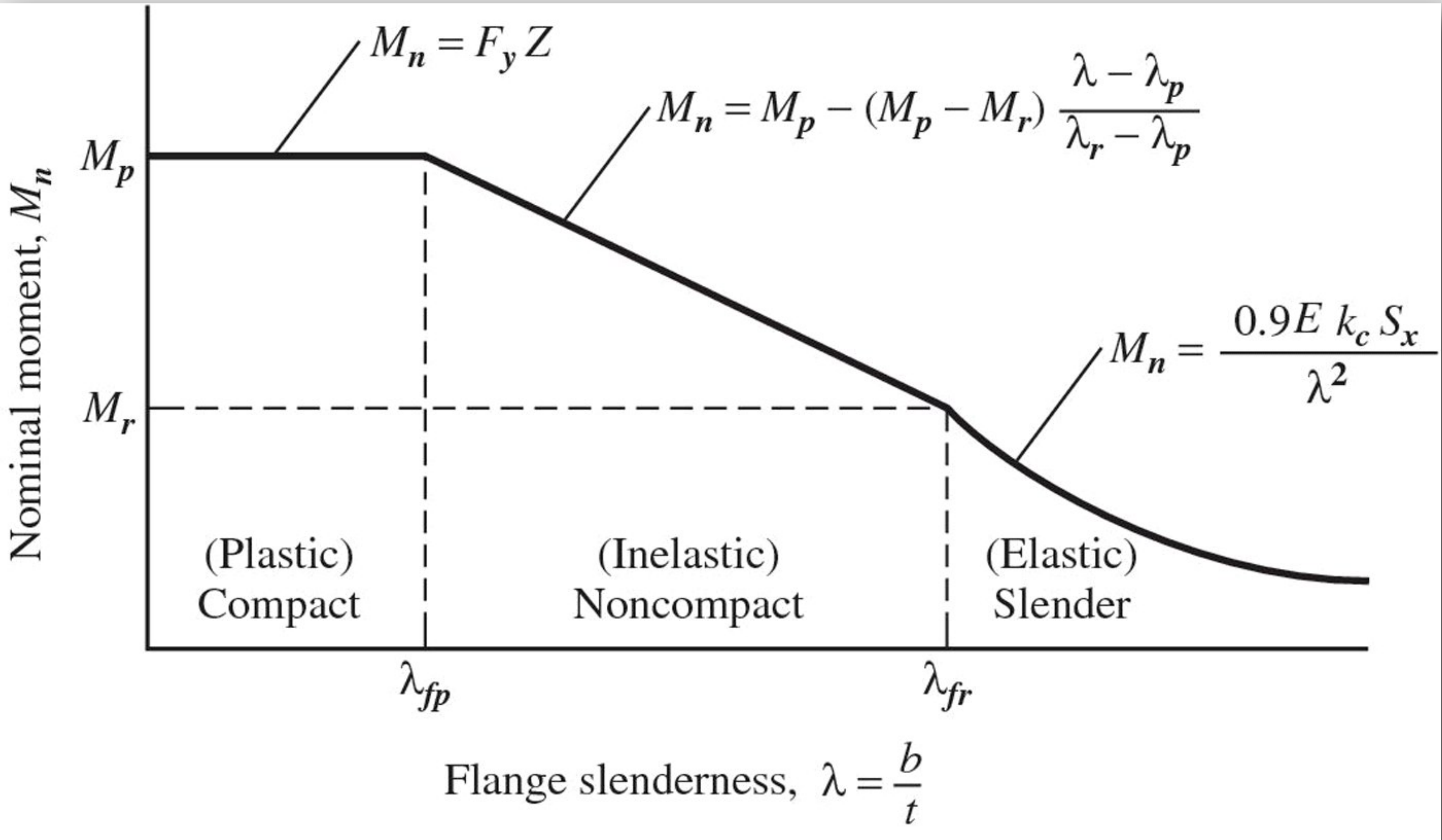


Figure 6.18

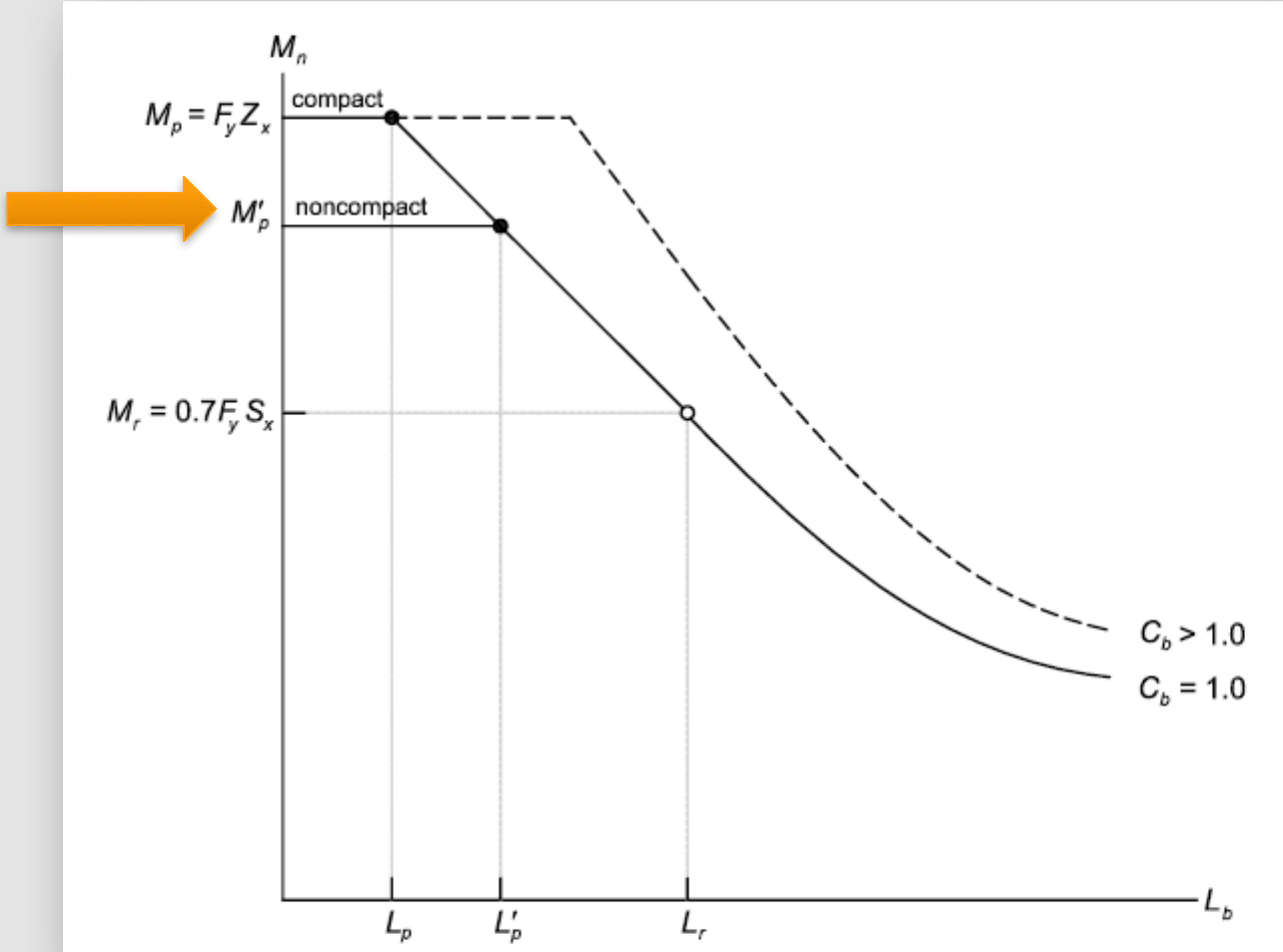
Maximum Flexural Capacity for Noncompact Sections

- Maximum moment capacity limited, regardless of LTB capacity:
  - For WF beam with noncompact flanges:

$$M_n = M_p - \left(M_p - 0.7F_y S_x\right)\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}$$

  - WF beams with slender flanges, or noncompact/slender webs trigger additional checks. (Not covered in this class).

Effect of Noncompact Section on Moment Capacity

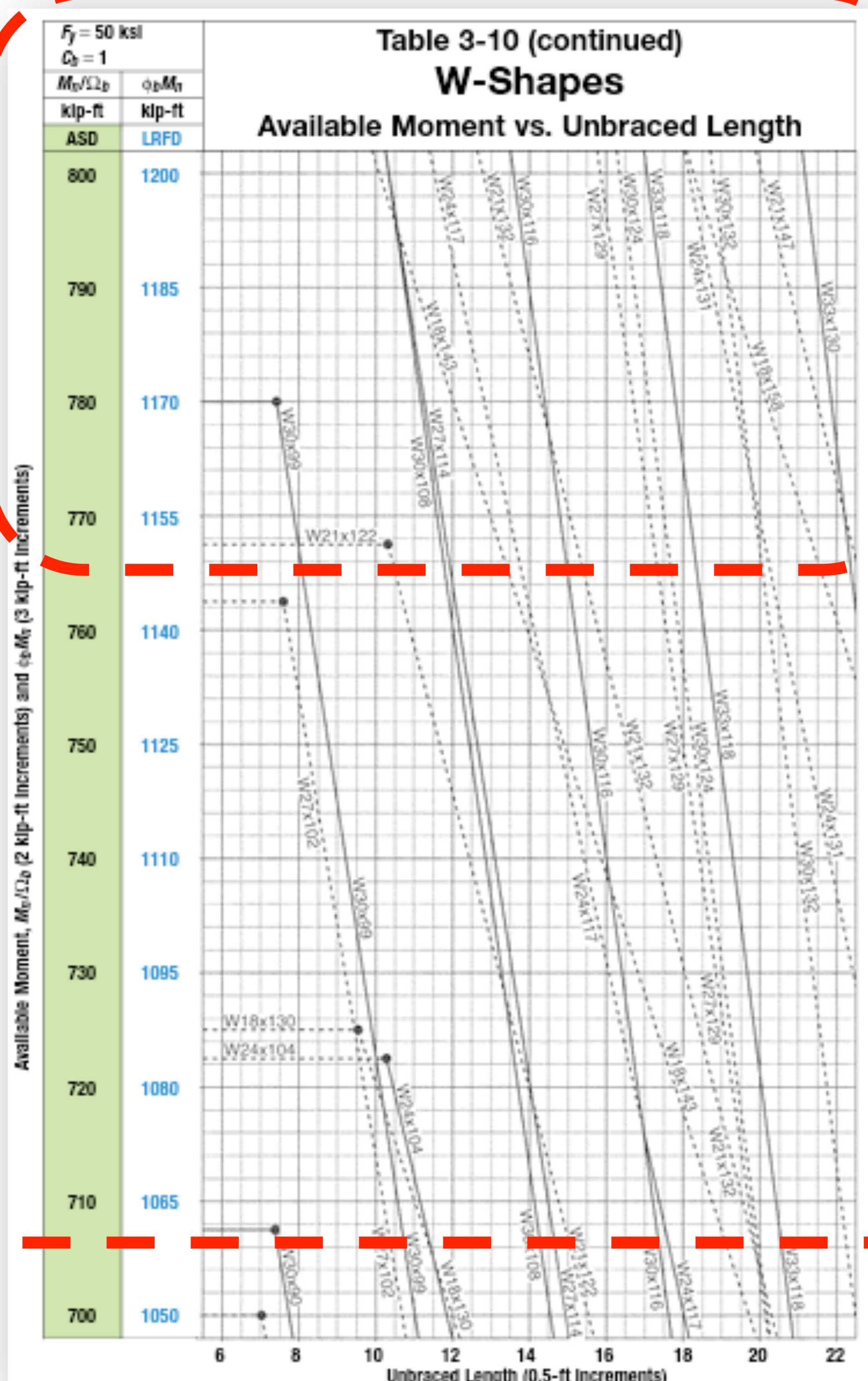


Design Checks for Flexure

1. Lateral Torsional Buckling
   - Checked for adequate lateral support of the compression flange.
2. Flange Local Buckling
   - Checked with $b_f/2t_f$ ratio.
3. Web Local Buckling
   - Checked with h/t_w ratio.
4. Formation of Plastic Moment
   - $M_u \leq \phi_b M_p$

Do We Need All These Equations?

- Refer to AISC
  - Section Properties
    - Table 1-1: width-thickness ratios, S, I, Z
  - Beam Tables
    - Table 3-2: select most economical size based on Z_x
    - Table 3-3: select most economical size based on I_x
    - Table 3-10: select most economical size for particular unbraced length
      - Note: compactness of section is considered in tables



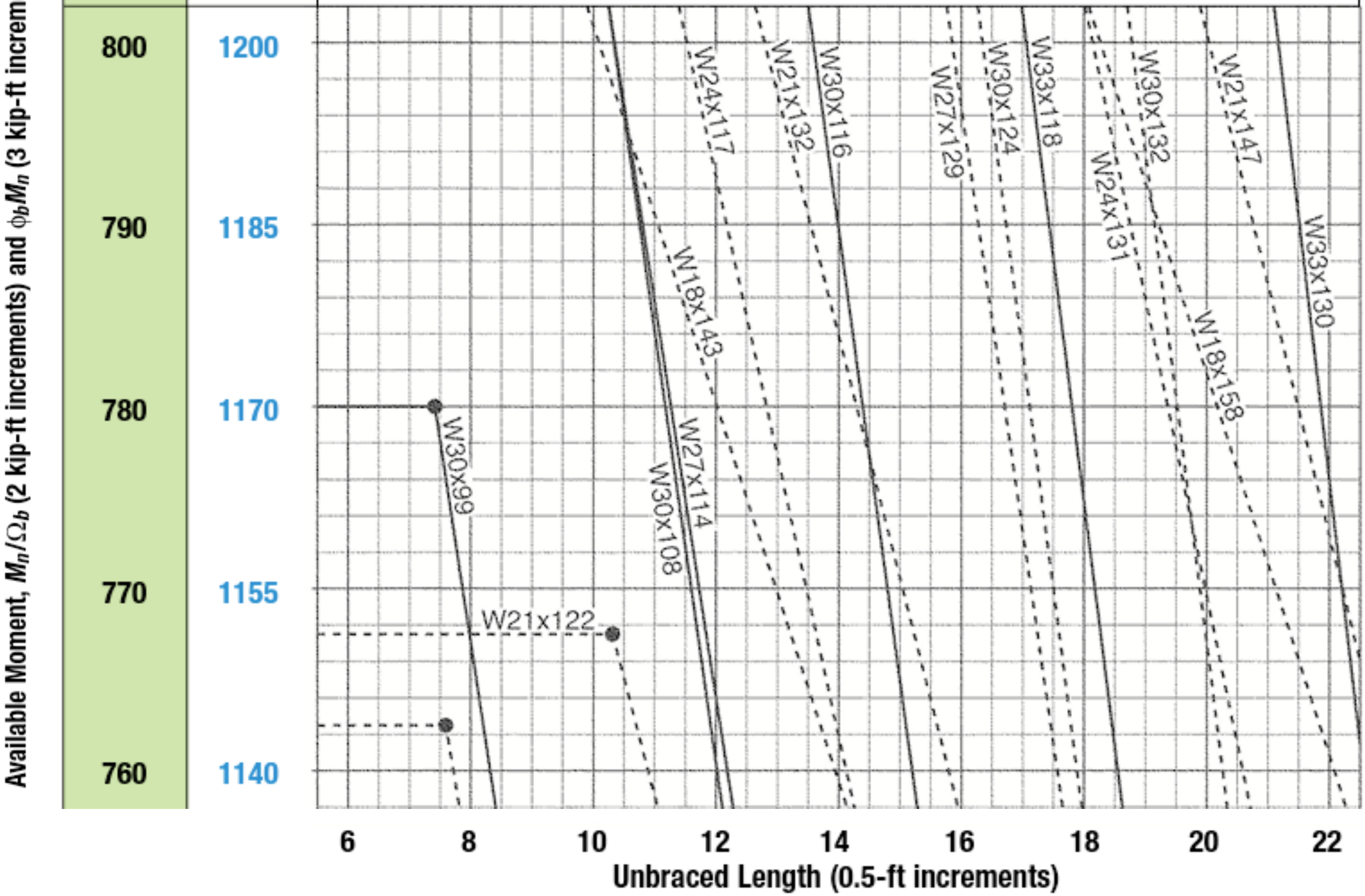
Beam Tables Based on Strong Axis Strength & Unbraced Length

- Actual unbraced length (L_b) of the beam is accounted for
- Very commonly used table

$F_y = 50$ ksi
$C_b = 1$

M_n/Ω_b	$\phi_b M_n$
kip-ft	kip-ft
ASD	LRFD

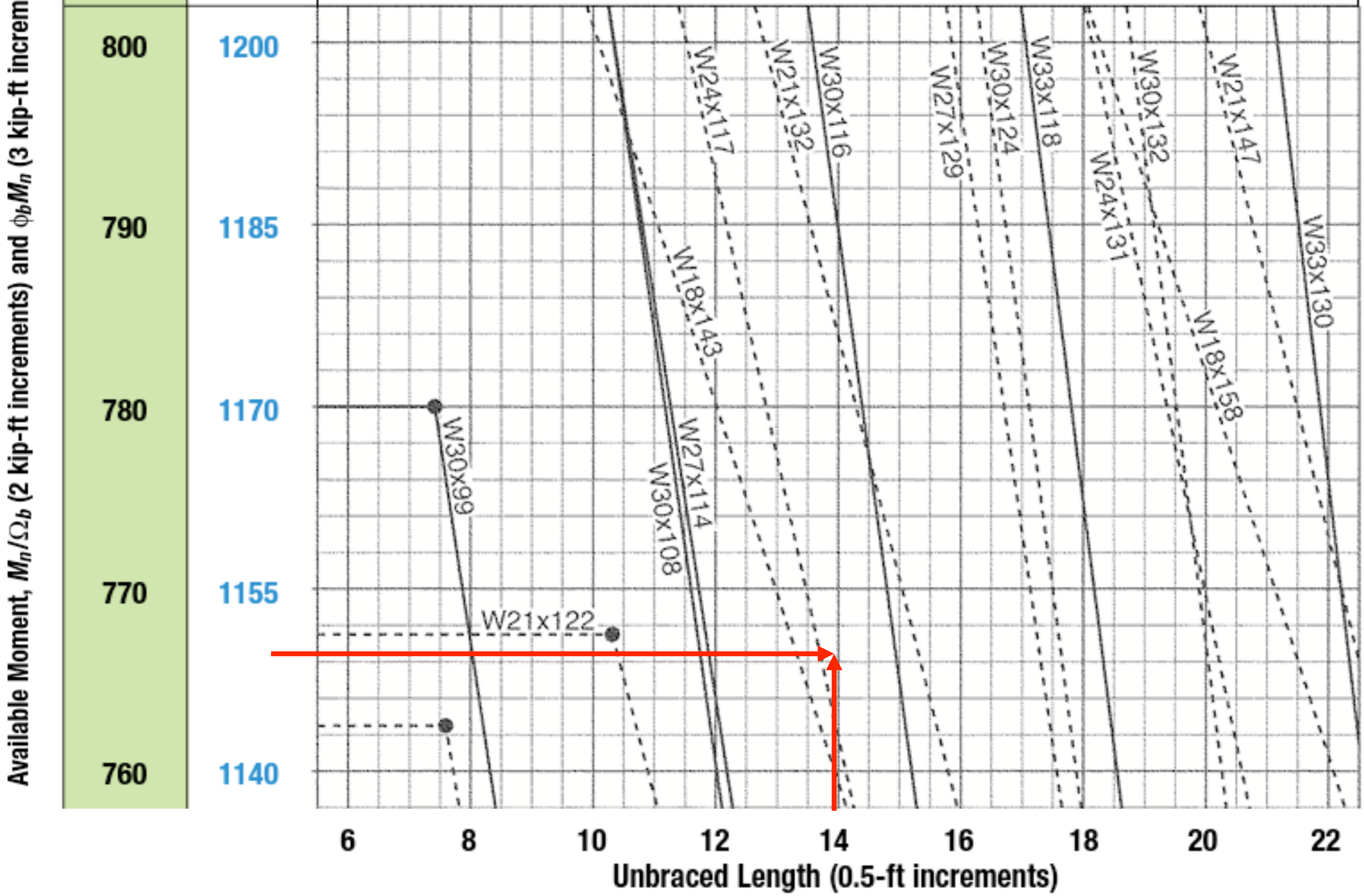
Table 3-10 (continued)
W-Shapes
Available Moment vs. Unbraced Length



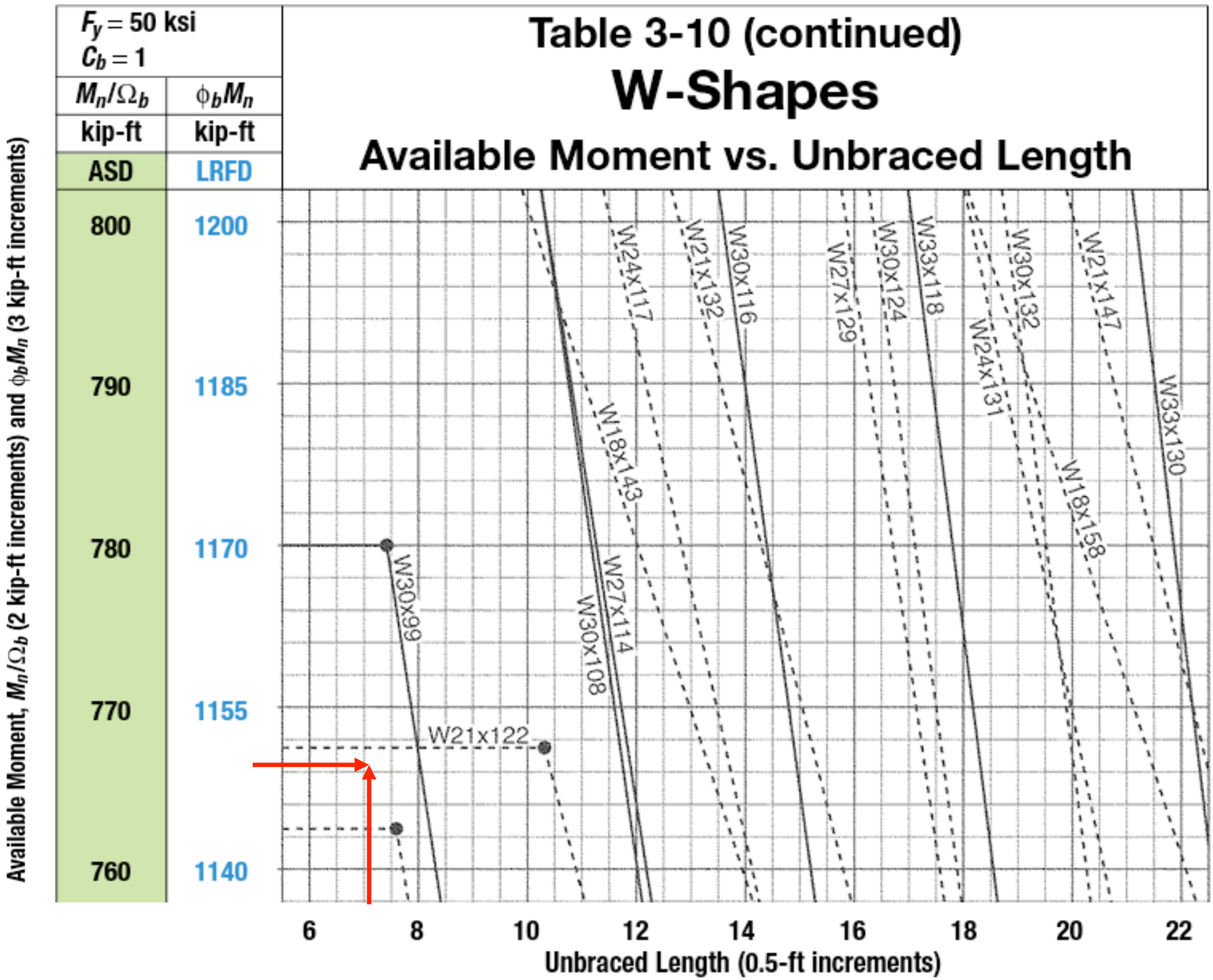
EXAMPLE: Lb= 14 ft, Mu = 1150 kft, Cb = 1.0

$F_y = 50$ ksi $C_b = 1$	
M_n/Ω_b	$\phi_b M_n$
kip-ft	kip-ft
ASD	LRFD

Table 3-10 (continued)
W-Shapes
Available Moment vs. Unbraced Length

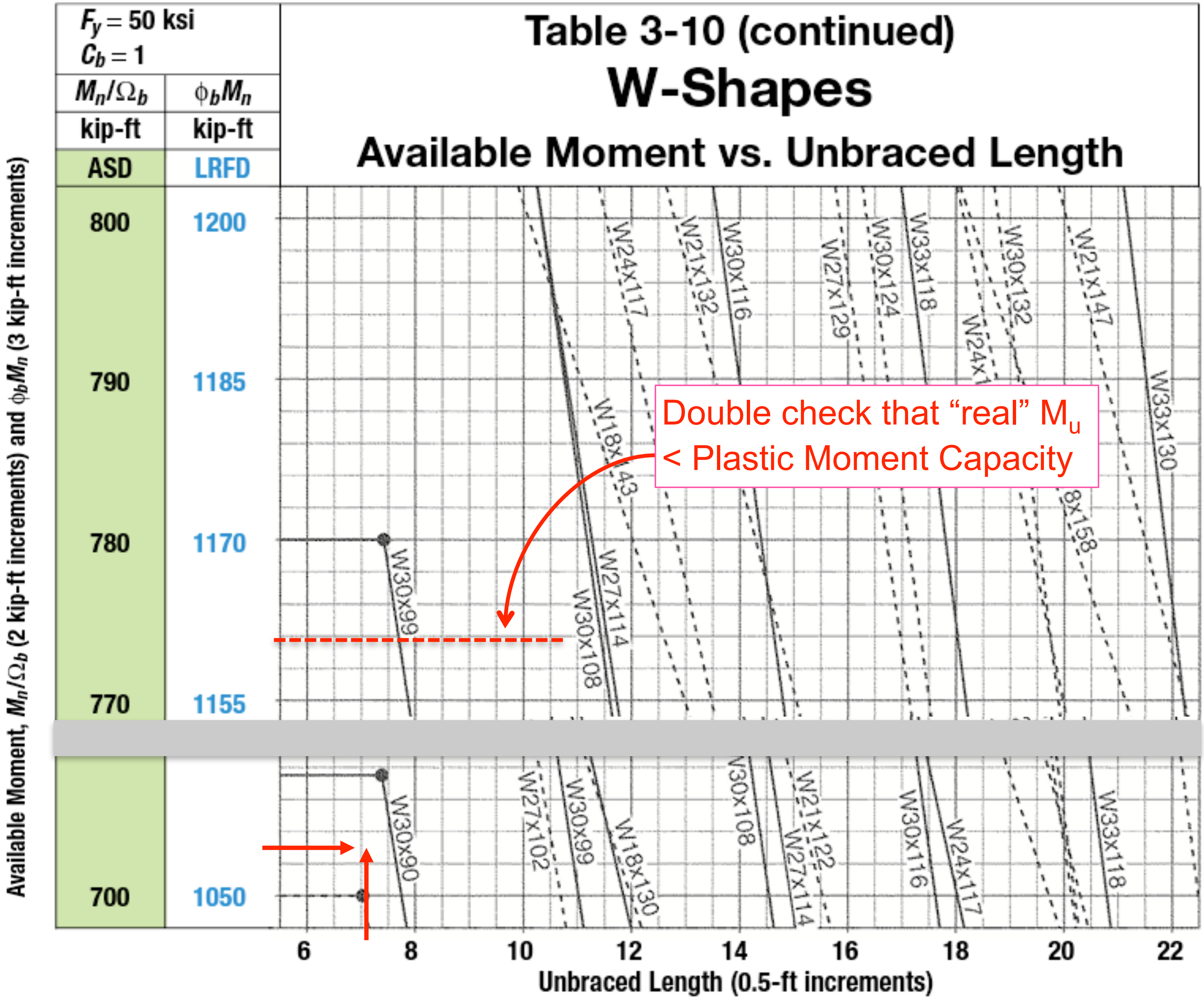


EXAMPLE: Lb= 7 ft, Mu = 1150 kft, Cb = 1.0



EXAMPLE: Lb= 7 ft, Mu = 1160 kft, Cb = 1.10

“Fictional” Mu = Mu/Cb = 1160/1.1 = 1055 kft

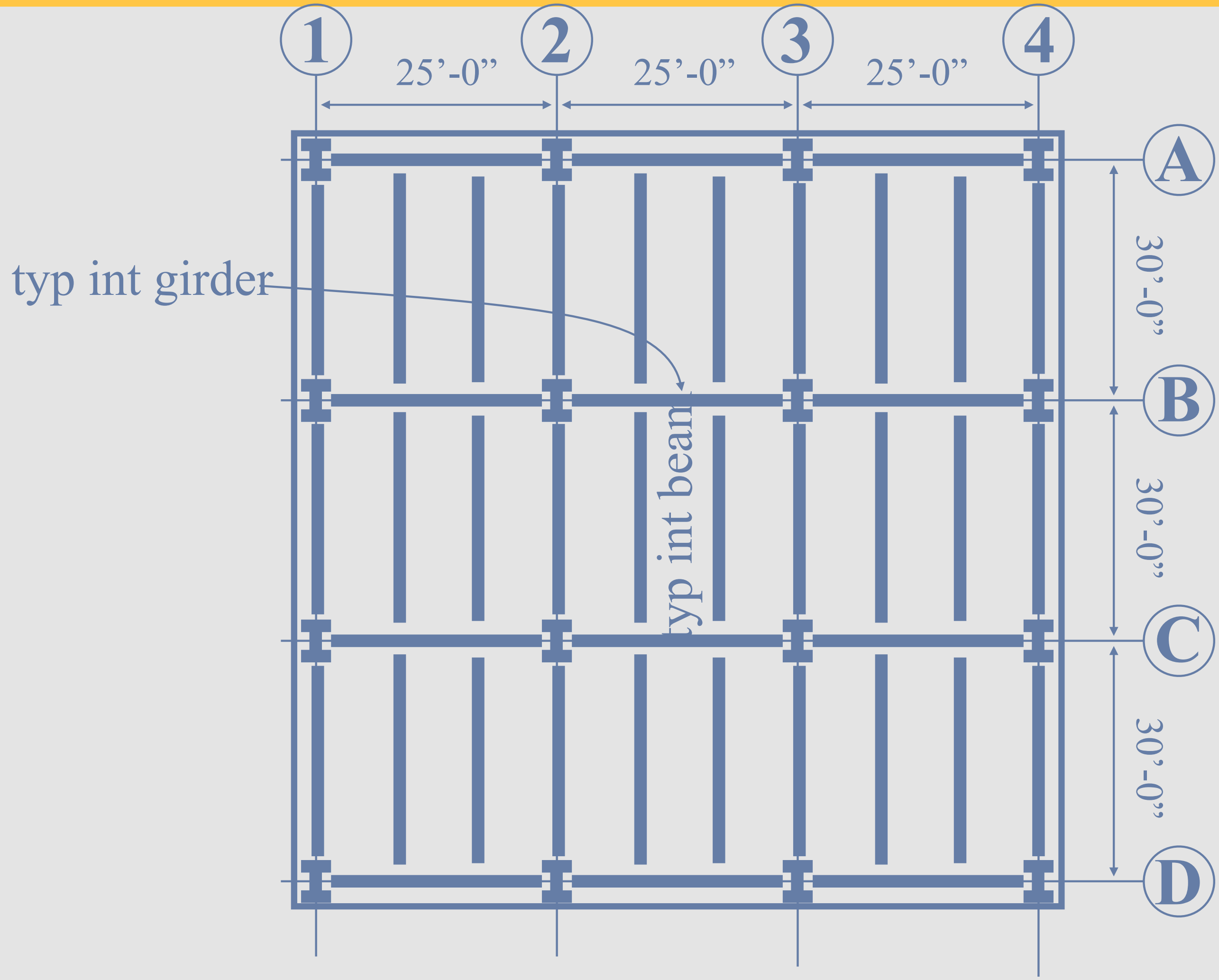


Questions?

Example Problem

- Design the most efficient typical interior WF beam.
- Design the most efficient typical interior WF girder.
- Assumptions:
  - Floor Plans are on the following page
  - Floor Loading: Dead = 100psf, Live = 40 psf (reducible)
  - Self weight of the members is included in the dead load
  - A992 Steel
  - Beam compression flanges are unbraced
  - Girder compression flanges are braced only at beam connection locations
  - Deflection Limits: Dead + Live = L/240, Live = L/360
  - Live Load Reductions: $L = L_0[0.25+15/\text{sqrt}(K_{LL}\,A_T)]$, $L \geq 0.50L_0$ (ASCE 7-05 4.8)

Example Problem



Floor Framing Plan